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Orientifold planes, type-I Wilson lines and non-BPS D-branes

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Abstract: There is a longstanding puzzle concerned with the existence of $\widetilde{\mathrm{O}p}^-$-planes with $p \geq 6$, which are 0orientifold p-planes of negative charge with stuck Dp-branes. We study the consistency of configurations with various orientifold planes and propose a resolution to this puzzle. It is argued that $\widetilde{\mathrm{O6}}^-$-planes are possible in massive IIA theory with odd cosmological constant, while $\widetilde{\mathrm{O7}}^-$-planes and $\widetilde{\mathrm{O8}}^-$-planes are not allowed. Various relations between orientifold planes and non-BPS D-branes are also addressed.

Keywords: Superstrings and Heterotic Strings, D-branes, Anomalies in Field and String Theories.

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Contents

1. Introduction

In type-II string theory there are perturbative $\mathbb{Z}_2$ symmetries with orientation reversal on the world-sheet of fundamental strings. And by keeping only states invariant under these $\mathbb{Z}_2$ symmetries, we can construct consistent theories with fixed planes called orientifold planes. An orientifold plane extended in p spatial directions (and one timelike direction) is denoted by Op-plane.

The manifold enclosing an Op-plane is $\mathbb{RP}^{8-p}$ and Op-planes are classified by the cohomology groups of this manifold [1, 2, 3]. As an example, let us consider an O5-plane. An O5-plane is surrounded by $\mathbb{RP}^3$, which has the following non-trivial integral cohomology groups:

$$H^3(\mathbb{RP}^3,\mathbb{Z}) = \mathbb{Z}\,, \qquad H^3(\mathbb{RP}^3,\widetilde{\mathbb{Z}}) = \mathbb{Z}_2\,, \qquad H^1(\mathbb{RP}^3,\widetilde{\mathbb{Z}}) = \mathbb{Z}_2\,, \tag{1.1}$$

where $\widetilde{\mathbb{Z}}$ is a "twisted" sheaf of integers. The first one is associated with the R-R charge carried by the O5-plane, i.e. the number of D5-branes coinciding it. The second one is the discrete torsion associated with the NS B-field and this determines the sign of R-R charge of the O5-plane itself. The unit element and the non-trivial element of $H^3(\mathbb{RP}^3,\widetilde{\mathbb{Z}})$ correspond to the O5-plane with the negative and the positive R-R charge, respectively. We will represent this charge by superscript like O5$^{\pm}$. The

	$\mathrm{O}p^-$	$\mathrm{O}p^+$	$\widetilde{\mathrm{O}p}^-$	$\widetilde{\mathrm{O}p}^+$
R-R charge	$n-2^{p-5}$	$n+2^{p-5}$	$n+1/2-2^{p-5}$	$n+2^{p-5}$
gauge group	SO$(2n)$	USp$(2n)$	SO$(2n+1)$	USp$(2n)$

Table 1: Properties of Op-planes for $p \leq 5$.

third one is the discrete torsion associated with the R-R 0-form field (the axion field). We use a notation $\widetilde{\mathrm{O5}}$ for O5-planes associated with a non-trivial element of this $\mathbb{Z}_2$ discrete torsion.

Similarly, for Op-planes with $p \leq 5$, there are two $\mathbb{Z}_2$ discrete torsions associated with the NS B-field and the R-R $(5-p)$-form field. Therefore, there are (at least) four kinds of orientifold p-planes, O$p^{\pm}$ and $\widetilde{\mathrm{O}p}^{\pm}$. The R-R charges and the gauge groups which appear on these orientifold planes with n Dp-branes are as in table 1. This list implies that an $\widetilde{\mathrm{O}p}^-$-plane can be interpreted as an Op^--plane with a half Dp-brane stuck on it.

For $p \geq 6$, however, we have no $\mathbb{Z}_2$ torsion associated with R-R fields. This seems to imply that there are only two kinds of orientifold planes O$p^{\pm}$. At first sight, it seems to be possible to construct an $\widetilde{\mathrm{O}p}^-$-plane by considering a half Dp-brane stuck on an Op^--plane. However, there are arguments from gauge theories on probe D-branes which exclude the existence of $\widetilde{\mathrm{O6}}^-$-planes and $\widetilde{\mathrm{O7}}^-$-planes. First, let us consider an $\widetilde{\mathrm{O7}}^-$-plane. The theory on one probe D3-brane near the O7$^-$-plane is $\mathcal{N}=2$ SU(2) gauge theory. The number of complex fermion doublets is twice the number of background D7-branes. If there is a stuck D7-brane on the O7$^-$-plane, which is counted as 1/2 D-brane, the number of fermion doublets is odd and the field theory suffers from the Witten's anomaly [4]. Therefore, a stuck D7-brane on an O7$^-$-plane, i.e. an $\widetilde{\mathrm{O7}}^-$-plane, is not allowed.

For an $\widetilde{\mathrm{O6}}^-$-plane, a relevant anomaly is the parity anomaly [5, 6] on a probe D2-brane. In three-dimensional SU(2) gauge theory, a fermion 1-loop induces the following Chern-Simons term for each complex fermion doublet.

$$\Gamma = \frac{1}{8\pi}\,\mathrm{sign}(m)\int_3 \mathrm{tr}\left(A\wedge dA + \frac{2}{3}A\wedge A\wedge A\right) = \frac{1}{8\pi}\,\mathrm{sign}(m)\int_4 \mathrm{tr}(F\wedge F)\,, \quad (1.2)$$

where m is a real fermion mass and the second integral is taken over a four-manifold whose boundary is the three-dimensional spacetime. By a large gauge transformation which changes the instanton number in the four-manifold by one, the effective action is changed by

$$\Delta\Gamma = 2\pi\frac{\mathrm{sign}(m)}{2}\,. \quad (1.3)$$

If the number of fermion doublets is odd, this large gauge transformation is broken regardless of the fermion masses. This seems to prohibit the existence of a stuck D6-brane on an O6$^-$-plane, i.e. an $\widetilde{\mathrm{O6}}^-$-plane.

From these facts, one may jump at the conclusion that $\widetilde{\mathrm{O}p}^-$-planes for $p \geq 6$ do not exist. As we will see later, this observation is partially correct. The situation, however, is not so simple. For example, considering T-duality, we immediately come up against a puzzle. That is, because there are both O5-planes and $\widetilde{\mathrm{O5}}^-$-planes, it seems possible to construct an $\widetilde{\mathrm{O6}}^-$-plane by T-duality from a pair of O5 and $\widetilde{\mathrm{O5}}^-$. If this is true, we would also be able to construct an $\widetilde{\mathrm{O7}}^-$-plane as a T-dual configuration of a pair of O6 and $\widetilde{\mathrm{O6}}^-$.

In this paper, we analyze consistency of orientifold configurations and propose a resolution of this puzzle. In particular, we will show that $\widetilde{\mathrm{O6}}^-$-planes do exist, while $\widetilde{\mathrm{O7}}^-$-planes and $\widetilde{\mathrm{O8}}^-$-planes do not. Interestingly, the absence of $\widetilde{\mathrm{O7}}^-$ and $\widetilde{\mathrm{O8}}^-$ is related to the cancellation of $\mathbb{Z}_2$ charges of non-BPS D7-branes and non-BPS D8-branes in type-I string theory. Actually, some arguments related to non-BPS D-branes turn out to be useful in our analysis.

We mainly focus our attention on $\mathrm{O}p^-$ and $\widetilde{\mathrm{O}p}^-$, though we also obtain several new results concerned with the other types of orientifold planes.

The outline of this paper is as follows. In section 2 we analyze Wilson lines in toroidally compactified type-I string theory, and give some rules to obtain consistent Wilson lines. The constraints for the consistent configurations with $\widetilde{\mathrm{O}p}^-$-planes are related to these rules by T-duality. We will show in section 3 that the results in section 2 are actually consistent under T-duality, with paying attention to the puzzle mentioned above. Section 4 is devoted to develop general arguments about orientifolds. We interpret discrete torsions of orientifolds by using spherical D-branes and NS-branes wrapped around orientifold planes. In section 5 we will make several comments about the relations between non-BPS D-branes and orientifolds. One of the observation given in section 4 and 5 is that we can continuously take off stuck Dp-branes from $\widetilde{\mathrm{O}p}$-planes in some configurations. We further confirm this phenomenon in section 6 from the viewpoint of the corresponding Wilson lines in type-I string theory.

2. Wilson lines in type-I string theory

In this section, we will analyze Wilson lines in toroidally compactified type-I string theory. Before discussing Wilson lines, we have to clarify the global structure of the gauge group in type-I string theory. Locally, gauge group is isomorphic to Spin(32). The center of Spin(32) is $\mathbb{Z}_2^L \times \mathbb{Z}_2^R$ and the vector, the spinor and the conjugate spinor representations carry the central charge $(-,-)$, $(-,+)$ and $(+,-)$, respectively. All fields in perturbative type-I string theory are neutral with respect to this center because an open string always has two end points belonging to the vector representation. Therefore, the perturbative gauge group is $\mathrm{Spin}(32)/(\mathbb{Z}_2^L \times \mathbb{Z}_2^R) = \mathrm{SO}(32)/\mathbb{Z}_2$. Furthermore, we can enlarge this gauge group to $\mathrm{O}(32)/\mathbb{Z}_2$, since transformations of determinant -1 are also symmetries in type-I perturbative string theory. Note that elements of O(32) with determinant -1 exchange representations with central charge $(-,+)$ and those with $(+,-)$. The perturbative spectrum of type-I string theory is invariant under this operation.

Non-perturbatively, however, the situation becomes different. Recently, it was found that type-I string theory has stable non-BPS D-instantons and stable non-BPS D-particles in its spectrum [7, 8]. The existence of these objects suggests that the non-perturbative gauge group of type-I string theory is $\mathrm{Spin}(32)/\mathbb{Z}_2^R$, as expected from the type-I heterotic duality. Actually, the non-BPS D-particles belong to the spinor representation with central charge $(-,+)$, which is not invariant under $\mathbb{Z}_2^L$.

Let us argue Wilson lines of this non-perturbative gauge group. We will discuss only Wilson lines with vector structure. This implies that dual orientifolds contain only $\mathrm{O}p^-$ and $\widetilde{\mathrm{O}p}^-$ [9]. $\mathrm{O}p^+$ and $\widetilde{\mathrm{O}p}^+$ are not considered in this section. Instead, thanks to the vector structure, we can diagonalize all Wilson lines simultaneously on the vector representation. Furthermore, to make our argument simple, we assume all diagonal elements of Wilson lines are $+1$ or -1. The incorporation of diagonal elements of generic values does not change the arguments below.

First, let us consider type-I string theory compactified on S^1 with a Wilson line g_1. Here the subscript of g denotes the compactified direction. Because g_1 has to be an element of $\mathrm{Spin}(32)/\mathbb{Z}_2^R$, the following condition should hold.

Condition A. *The number of -1 components of the Wilson line is even.*

For example, the following Wilson line is not allowed.

$$g_1 = \mathrm{diag}\left(-1, +1, (+1)^{30}\right). \tag{2.1}$$

If we allow this Wilson line, it causes an inconsistency that the chirality of the non-BPS D-particle flips when it goes around the compactified direction, because g_1 is represented by $\Gamma\Gamma^1$ on the spinor representation. Here Γ^a are gamma matrices of SO(32) and $\Gamma = \Gamma^1 \cdots \Gamma^{32}$. It is well known that Wilson lines correspond to the positions of D-branes in the T-dual picture. Therefore, the T-dual configuration of the Wilson line (2.1) has two $\widetilde{\mathrm{O}8}^-$-planes. So forbiddance of the Wilson line (2.1) implies that $\widetilde{\mathrm{O}8}^-$-planes in the T-dual picture are not allowed.

The Wilson line (2.1) corresponds to the non-trivial element of $\pi_0(\mathrm{O}(32)) = \mathbb{Z}_2$ and is studied in many works in connection with non-BPS D8-branes. Although it would be possible to construct any number of eight-branes perturbatively, the $\mathbb{Z}_2$ charge should be cancelled on S^1 in order to be compatible with the existence of non-BPS D-particles, as explained above. We will reconsider this point in section 5.

Let us move on to $\mathbb{T}^2$ compactification of type-I string theory and consider the following Wilson lines:

$$\begin{aligned} g_1 &= \mathrm{diag}\left(-1, +1, -1, +1, (+1)^{28}\right), \\ g_2 &= \mathrm{diag}\left(-1, -1, +1, +1, (+1)^{28}\right). \end{aligned} \tag{2.2}$$

If these Wilson lines are possible, it would give four $\widetilde{\mathrm{O7}}^-$-planes as a T-dual configuration. However, we can again show this is not allowed as follows.

In general, Wilson lines for any two directions should commute. More precisely, they should commute on the representation of any matter field in the theory. In type-I string theory, since the non-BPS D-particle belongs to the spinor representation, g_1 and g_2 should commute on the spinor representation. The spinor representations of these group elements are

$$g_1 = \Gamma^1\Gamma^3\,, \qquad g_2 = \Gamma^1\Gamma^2\,. \tag{2.3}$$

Clearly, these two matrices do not commute with each other. (Instead, they anti-commute.) Therefore, these Wilson lines are not allowed and we cannot make $\widetilde{\mathrm{O7}}^-$-planes as a T-dual configuration.

Topologically, the Wilson lines (2.2) correspond to a non-trivial element of homotopy group $\pi_1(\mathrm{O}(32)) = \mathbb{Z}_2$ which corresponds to a non-BPS D7-brane. The argument above implies that the $\mathbb{Z}_2$ charge carried by non-BPS D7-branes must be cancelled on $\mathbb{T}^2$, as in the case of non-BPS D8-branes. See section 5 for more details.

In order for any two Wilson lines to commute, we must impose the following condition in addition to the condition A.

> **Condition B.** *For any two g_i and g_j, the number of components which are -1 for both g_i and g_j is even.*

This statement is obtained by using the fact that the spinor representation of g_i is given as a product of all gamma matrices having indices of -1 components. (Thanks to condition A, Γ insertion is not necessary.)

Let us proceed to the case of $\mathbb{T}^3$ compactification. We can make the following Wilson lines:

$$\begin{aligned} g_1 &= \mathrm{diag}\left(-1,+1,-1,+1,-1,+1,-1,+1,(+1)^{24}\right),\\ g_2 &= \mathrm{diag}\left(-1,-1,+1,+1,-1,-1,+1,+1,(+1)^{24}\right),\\ g_3 &= \mathrm{diag}\left(-1,-1,-1,-1,+1,+1,+1,+1,(+1)^{24}\right). \end{aligned} \tag{2.4}$$

These Wilson lines satisfy the two conditions A and B and hence are compatible with the non-perturbative gauge group. We claim that these Wilson lines are really possible and the T-dual configuration with eight $\widetilde{\mathrm{O6}}^-$-planes is as well. This seems to be inconsistent with the argument of discrete torsions. We will discuss this point in the next section.

Although we can construct Wilson lines dual to eight $\mathrm{O6}^-$ or eight $\widetilde{\mathrm{O6}}^-$, there are no Wilson lines dual to mixed configurations containing both $\mathrm{O6}^-$ and $\widetilde{\mathrm{O6}}^-$. The proof is as follows. On the orientifold $\mathbb{T}^3/\mathbb{Z}_2$, there are eight O6-planes. We normalize their positions on the $\mathbb{T}^3/\mathbb{Z}_2$ as $(\pm1,\pm1,\pm1)$, i.e. as apexes of a cube with

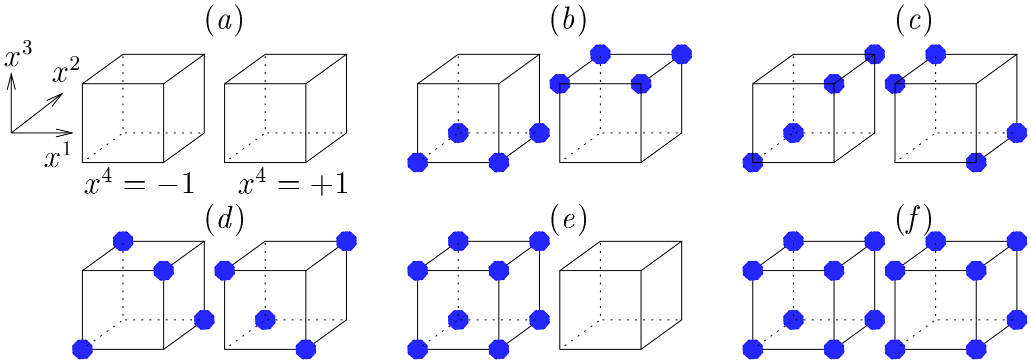

Figure 1: Each possible configuration of O5-planes is the same with one of these up to rotation. The cubes spread in the x^1, x^2, x^3 directions and left (right) ones are located at $x^4 = -1$ $(+1)$. O5-planes are located at the apexes of the cubes, and the bullets represent $\widetilde{\mathrm{O5}}^-$-planes. We have 1 (a)-type, 12 (b)-type, 8 (c)-type, 2 (d)-type, 8 (e)-type and 1 (f)-type configurations. (b), (c), (d) and (e) are equivalent to each other via modular transformations.

sides 2. The two conditions above demand two adjacent apexes to be the same type of orientifold planes. And this results that all apexes must be the same type of O6-planes. At first sight, this may seem strange because this implies there is some correlation among eight O6-planes. In the next section, we will see this is explained in a natural way in the dual orientifold picture.

For a $\mathbb{T}^4/\mathbb{Z}_2$ orientifold, 16 O5-planes are put on each apex of a four-dimensional cube and the two conditions above demand the number of $\widetilde{\mathrm{O5}}^-$ on each (two-dimensional) face should be even: 0, 2 or 4. Let α denote an apex of the four-dimensional cube and $P_i\alpha$ $(i = 1, 2, 3, 4)$ denote its reflection along the i-th axis. If we introduce $\sigma(\alpha)$ which is $+1$ (-1) for α corresponding to $\mathrm{O5}^-$ $(\widetilde{\mathrm{O5}}^-)$, the conditions above are equivalent to the statement that the following value does not depend on the apex α.

$$c_i = \sigma(P_i\alpha)\sigma(\alpha)\,. \tag{2.5}$$

Therefore, general solution for the conditions is represented as follows:

$$\begin{aligned}
\sigma(P_i\alpha) &= c_i\sigma(\alpha)\,,\\
\sigma(P_iP_j\alpha) &= c_ic_j\sigma(\alpha)\,, \quad (i<j)\,,\\
\sigma(P_iP_jP_k\alpha) &= c_ic_jc_k\sigma(\alpha)\,, \quad (i<j<k)\,,\\
\sigma(P_1P_2P_3P_4\alpha) &= c_1c_2c_3c_4\sigma(\alpha)\,,
\end{aligned} \tag{2.6}$$

where α in these equations is fixed. Because we can freely choose five variables, c_i and $\sigma(\alpha)$, there are 32 allowed configurations. It is easy to construct these 32 possible configurations explicitly, and we find that the number of $\widetilde{\mathrm{O5}}^-$ is restricted to be one of 0, 8 and 16. (Figure 1)

On a $\mathbb{T}^n/\mathbb{Z}_2$ orientifold, there are 2^n O$(9-n)$-planes. They are labeled by n-dimensional vectors with components ± 1. Let us define a subset of these O$(9-n)$-planes whose labels have $(n-3)$ or less $+1$ components. This subset contains $\sum_{k=0}^{n-3} {}_nC_k$ O$(9-n)$-planes, and we can freely choose the types of them. The types of the other O-planes are uniquely determined by the two conditions. Therefore, the number of possible configurations is

$$N = 2^{\sum_{k=0}^{n-3} {}_nC_k} = 2^{2^n-(n^2+n+2)/2} \,. \tag{2.7}$$

When $n \geq 6$, the number of orientifold planes is larger than 32 and some of N configurations should contain anti D-branes so that the total R-R charge is cancelled. Because we have not used the supersymmetry, our argument is applicable even in such cases.

We have obtained two necessary conditions for the Wilson lines to be allowed. We expect that these conditions are sufficient to select possible Wilson lines. However, it is not a simple task to prove this statement. In the next section, we will discuss T-duality among orientifold planes with different dimensions. We will show the argument is very consistent with the two conditions we obtained in this section. It strongly suggests the conditions are sufficient.

3. T-duality of orientifold planes

Having analyzed Wilson lines on torus in type-I string theory, some questions arise. First, we saw that the conditions for the possible Wilson lines in type-I string theory suggests the existence of both $\mathrm{O6}^-$ and $\widetilde{\mathrm{O6}}^-$. Is this consistent with the analysis of the discrete torsion and the anomaly mentioned in section 1? Next, we saw that we cannot freely choose the types of orientifold planes in a compact orientifold. How does this correlation among orientifold planes explained? Finally, how is it prohibited to make $\widetilde{\mathrm{O7}}^-$ or $\widetilde{\mathrm{O8}}^-$ by T-duality?

To answer these questions, let us consider T-duality between an $\widetilde{\mathrm{O6}}^-$-plane and a pair of $\mathrm{O5}^-$ and $\widetilde{\mathrm{O5}}^-$.

$$\mathrm{O5}^- + \widetilde{\mathrm{O5}}^- \longleftrightarrow \widetilde{\mathrm{O6}}^- \,. \tag{3.1}$$

Here we assume the positions of the $\mathrm{O5}^-$-plane and $\widetilde{\mathrm{O5}}^-$-plane to be $(x^1,x^2,x^3,x^4) = (0,0,0,0)$ and $(0,0,0,a)$ respectively. The $\mathbb{Z}_2$ actions for these orientifold planes are

$$\Omega : (x^1,x^2,x^3,x^4) \longrightarrow (-x^1,-x^2,-x^3,-x^4)\,, \tag{3.2}$$

$$\widetilde{\Omega} : (x^1,x^2,x^3,x^4) \longrightarrow (-x^1,-x^2,-x^3,2a-x^4)\,, \tag{3.3}$$

respectively. The composition of these transformations is a shift along x^4 by $2a$,

$$\widetilde{\Omega}\Omega : (x^1,x^2,x^3,x^4) \longrightarrow (x^1,x^2,x^3,x^4+2a)\,. \tag{3.4}$$

This implies x^4 direction is compactified with a period $2a$. The T-duality is taken along this direction.

The orientifold flip acts on R-R fields non-trivially. Especially, Ω and $\widetilde{\Omega}$ change the sign of the axion field $C/2\pi$. Taking account of the freedom of shifting by an integer, we obtain the following transformation low:

$$\Omega : C(x^4)/2\pi \longrightarrow n - C(-x^4)/2\pi\,, \qquad \widetilde{\Omega} : C(x^4)/2\pi \longrightarrow n' - C(2a - x^4)/2\pi\,. \quad (3.5)$$

(The dependence on the other coordinates is omitted.) From (3.5), the value of the axion field on top of an orientifold plane should be a half integer. Up to an integral shift, we have two physically distinct cases: $C/2\pi \in \mathbb{Z}$ and $C/2\pi \in \mathbb{Z} + 1/2$. In fact, this value represents the discrete torsion associated with the orientifold plane. Therefore, the integral and half odd integral axion field correspond to $\mathrm{O5}^-$ and $\widetilde{\mathrm{O5}}^-$ respectively. In our case this fact implies

$$n \in 2\mathbb{Z}\,, \qquad n' \in 2\mathbb{Z} + 1\,. \quad (3.6)$$

Now, let us see how the axion field is transformed under the parallel transport (3.4). The composition of (3.5) is

$$\widetilde{\Omega}\Omega : C(x^4)/2\pi \longrightarrow C(x^4 + 2a)/2\pi - (n' - n)\,. \quad (3.7)$$

Notice that the change of the axion field $(n' - n)$ is always an odd integer and does not vanish. This is represented by the following equation:

$$\frac{1}{2\pi} \oint dC = n' - n\,, \quad (3.8)$$

where the integral is taken along the S^1. Under the T-duality along the x^4 direction, the R-R one-form field strength on the left-hand side is transformed into R-R zero-form field strength, which is electric-magnetic dual to the ten-form field strength of the R-R nine-form field. Namely, the configuration with different kinds of O5-planes are transformed into "massive" type-IIA theory with odd cosmological constant.[1]

The generalization of the argument above to include the case of a pair of the same kind of O5 is straightforward and the generalized statement is as follows.

A pair of the same kind of O5-planes is transformed into $\mathrm{O6}^-$ *in the background with even cosmological constant, while a pair of different kinds of O5-planes is mapped into* $\widetilde{\mathrm{O6}}^-$ *in the odd background cosmological constant.*

Although both $\mathrm{O6}^-$ and $\widetilde{\mathrm{O6}}^-$ are possible, once the background cosmological constant is fixed, one of them is automatically chosen. Namely, $\mathrm{O6}^-$ ($\widetilde{\mathrm{O6}}^-$) are

[1]In this paper, the R-R zero-form field strength is often called "the cosmological constant", and normalized to be an integer. Actually, it is the square root of the cosmological constant in a conventional sense, since it induces the cosmological term $S \sim \int d^{10}x\sqrt{-g}(F_0)^2$. We hope this expression will not lead any confusion.

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possible only in the background with even (odd) cosmological constant. This is consistent with the absence of $\mathbb{Z}_2$ torsion. Instead of $\mathbb{Z}_2$ torsion, we have a non-trivial cohomology group $H^0(\mathbb{RP}^2, \mathbb{Z}) = \mathbb{Z}$ associated with the R-R 0-form field strength. Therefore, the element of this cohomology group gives an extra charge of the O6-plane, which is identified with the background cosmological constant. This naturally gives the reason why all O6-planes in the $\mathbb{T}^3/\mathbb{Z}_2$ orientifold belong to the same type. Because the background cosmological constant fixes the type of O6-planes, all O6-planes in a BPS configuration have to be the same type.

Taking account of the relation between the cosmological constant and the type of O6-planes, the parity anomaly problem mentioned in the section 1 can be solved. The action of massive D2-brane contains the following term [10].

$$S = \frac{\Lambda}{8\pi} \int \mathrm{tr} \left(A \wedge dA + \frac{2}{3} A \wedge A \wedge A \right), \tag{3.9}$$

where Λ is the integral background cosmological constant. This term supply the bare Chern-Simons term in the bare action and combined with the one-loop effect of world-volume fermions (1.2), the invariance under large gauge transformations is recovered.[2]

Now, we have shown that both kinds of O6-planes are possible and solved the puzzle about T-duality between O5-planes and O6-planes. The T-duality between O6 and O7 should be considered next. One may suspect that we can have a pair of $\mathrm{O6}^-$ and $\widetilde{\mathrm{O6}}^-$ in some non-BPS configuration with varying cosmological constant. If so, is it possible to obtain $\widetilde{\mathrm{O7}}^-$-planes via T-duality transformation,

$$\mathrm{O6}^- + \widetilde{\mathrm{O6}}^- \longleftrightarrow \widetilde{\mathrm{O7}}^- \ ? \tag{3.10}$$

As explained in section 1, The existence of the $\widetilde{\mathrm{O7}}^-$-plane causes Witten's anomaly on the D3-brane probe field theory. Furthermore, half D-brane charge due to the stuck D7-brane on the $\mathrm{O7}^-$-plane implies the non-integral monodromy associated with the axion field. This is physically unacceptable.

The resolution of this puzzle is as follows. If we wish to consider the configurations with both $\mathrm{O6}^-$ and $\widetilde{\mathrm{O6}}^-$, the two regions should be divided by odd number of D8-branes playing a role of domain walls. For example, let us assume $\widetilde{\mathrm{O6}}^-$ is enclosed spherically by a D8-brane. (This D8-brane is "spherical" on the covering space. After $\mathbb{Z}_2$ orientifolding it becomes $\mathbb{RP}^2$.) Actually, the non-trivial twisted homology group $H_2(\mathbb{RP}^2, \widetilde{\mathbb{Z}}) = \mathbb{Z}$ shows such a wrapping is possible. Spherical D-brane

[2] The trace in eq. (3.9) is taken on the $\mathbf{2k}$ representation of the $\mathrm{USp}(2k)$ gauge group. If the D2-branes are moved away from the orientifold plane, the gauge group is broken to $\mathrm{U}(k)$. If we use generators for the fundamental representation of this $\mathrm{U}(k)$, the coefficient in eq. (3.9) is doubled because $\mathbf{2k}$ is decomposed into $\mathbf{k} + \overline{\mathbf{k}}$ representation of $\mathrm{U}(k)$. Therefore, the argument above does not conflict with the existence of massive D2-branes in odd Λ backgrounds.

configurations are considered in [11] and [12] in the context of matrix theory and in Myers [13] showes that such configurations are stabilized in the non-zero R-R field strength background. (Of course, in our case, it does not have to be stabilized at all.) Recently, such configurations are used to analyze $\mathcal{N}=1$ gauge theory [14, 15]. As discussed in [15], in order to be consistent with the results in field theory, the Dp-brane charge carried by a spherical D(p+2)-brane enclosing an orientifold p-plane should be shifted by half, which is induced by the shifted flux quantization condition,

$$\frac{1}{2\pi}\oint_{\mathbb{RP}^2} f_2 \in \mathbb{Z}+\frac{1}{2}\,. \tag{3.11}$$

Here f_2 is the field strength of the gauge field on the spherical D($p+2$)-brane. This is also suggested by the argument of stringy anomaly in [16]. We will show the equation (3.11) in section 5, using an argument of tachyon condensation. Therefore, the total amount of the D6-brane charge of $\widetilde{\mathrm{O6}}^-$ enclosed by a spherical D8-brane is the same with that of O6$^-$ (up to integer). Therefore, when the spherical D8-brane shrinks to a point, the pair of O6$^-$ and $\widetilde{\mathrm{O6}}^-$ reduces to a pair of O6$^-$-planes! This results that the T-dual of the O6 configurations are always not $\widetilde{\mathrm{O7}}^-$ but O7$^-$.

Up to now, we have seen that our results on $\widetilde{\mathrm{O6}}^-$ and $\widetilde{\mathrm{O7}}^-$ in the last section is also explained in the dual orientifold picture by using the discrete torsions. In fact, we can show the two conditions A and B are equivalent to the requirement that the discrete torsions on the orientifold should be well-defined. On a $\mathbb{T}^n/\mathbb{Z}_2$ orientifold with $n \geq 4$, a section with three coordinates x^i, x^j and x^k fixed is $\mathbb{T}^{n-3}$. The integral of the R-R $(n-3)$-form field strength on this section should not depend on the position (x^i, x^j, x^k). For example, the integral at $(+1,+1,+1)$ and that at $(+1,+1,-1)$ must be the same. These two integrals are nothing but the sums of the discrete torsions of O($9-n$)-planes on the sections. Therefore, the requirement above implies the sum of $\mathbb{Z}_2$ discrete torsions of O($9-n$)-planes on a section with x^i and x^j fixed should be zero. This is equivalent to the conditions we obtained in section 2.

Before ending this section, we would like to give one more consistency check. Let us consider the Wilson lines (2.4) and take a T-duality transformation along the x^3 direction. This system consists of two O8$^-$-planes with four D8-branes on each of them. (We omit last 24 components in (2.4) because they do not affect the following arguments.) The gauge group on each world-volume of four D8-branes is (locally) Spin(4) and each has the following Wilson lines:

$$g_1 = \mathrm{diag}(-1,+1,-1,+1)\,, \qquad g_2 = \mathrm{diag}(-1,-1,+1,+1)\,. \tag{3.12}$$

This has the same structure with the Wilson lines (2.2). Recall that the Wilson lines (2.2) are unacceptable, since g_1 and g_2 in (2.2) do not commute on the spinor representation, to which non-BPS D-particles belong. Now we are in face of the same problem. Because (BPS) D-particles stuck on one of the O8$^-$-planes belong to spinor representation of the Spin(4) gauge group on the world-volume of four D8-branes, it seems that the Wilson lines (3.12) are not acceptable as the Wilson lines (2.2) are not. If so, in addition to the two conditions we have already discussed, we have to introduce some stronger constraint. Can it be true? If we should introduce such a constraint, a lot of configurations, including (2.4), would be prohibited and the arguments so far become almost meaningless.

Before solving this problem, let us restate the reason why the Wilson lines (3.12) lead to the problem in a more convenient form. Let us consider closed world-line C of a stuck D-particle on the world-volume of four D8-branes. If C wrapped around the x^2 direction, the contribution of the Spin(4) gauge field A^a to the partition function of the D-particle is

$$Z_A = \mathrm{tr}\left(g_2 P \exp \int_C A^a T_a\right), \tag{3.13}$$

where T_a is a generator of Spin(4) gauge group in the spinor representation. The insertion of g_2 is necessary due to the Wilson line (3.12) along the x^2-direction. If we move the cycle C to go around the cycle along x^1, the partition function changes as follows:

$$Z'_A = \mathrm{tr}\left(g_1 g_2 g_1^{-1} P \exp \int_C A^a T_a\right). \tag{3.14}$$

Because $g_1 g_2 g_1^{-1}$ is equal to $-g_2$ In the spinor representation, Z_A and Z'_A do not coincide. This is against the singlevaluedness of the partition function.

A key saving us from this situation is the fact that the dual O6-plane configuration of (2.4) always has odd cosmological constant. It implies that in the O8-plane configuration the R-R two-form field strength F_{12} is non zero and the total flux is odd integer. Taking account of the fractional charge of a stuck D-particle, the contribution of R-R 1-form field to the partition function is

$$Z_{RR} = \exp\left(\frac{i}{2}\oint_C C_1\right). \tag{3.15}$$

If we move the path C around x^1, it sweeps out the x^1-x^2 torus, and the partition function changes to Z'_{RR} satisfying

$$\frac{Z'_{RR}}{Z_{RR}} = \exp\left(\frac{i}{2}\oint_{\mathbb{T}^2} dC_1\right). \tag{3.16}$$

Because total flux is odd integer, the left-hand side of (3.16) is -1. This correctly cancels the contribution of the Wilson lines (3.12). In other words, the gauge group associated with stuck D-particles is not $\mathrm{Spin}(4) \times \mathrm{U}(1)$ but

$$G = (\mathrm{Spin}(4) \times \mathrm{U}(1))/\mathbb{Z}_2\,. \tag{3.17}$$

The Wilson lines (3.12) are not well-defined as a gauge bundle for Spin(4) but they are for the gauge group (3.17) if they are accompanied by appropriate R-R gauge flux.

Note that this trick is not applicable to the Wilson lines (2.2) because there is no R-R gauge field coupling to non-BPS D-particles in type-I string theory.

4. Discrete torsions and spherical branes

In the previous section, we observed that an $\widetilde{\mathrm{O6}}^-$-plane enclosed by a spherical D8-brane is equivalent to an $\mathrm{O6}^-$-plane. We can easily generalize this argument to the cases including other types of orientifold planes.

Our first claim is as follows.

> *We can change Op to $\widetilde{\mathrm{O}p}$ by wrapping it spherically with a D$(p+2)$-brane, and vice versa.*

This statement is meaningful for Op-planes of $p \leq 6$. Note that we have a non-trivial twisted 2-cycle $\mathbb{RP}^2$, on which the D$(p+2)$-brane is wrapped, as a submanifold of the $\mathbb{RP}^{8-p}$ enclosing the Op-plane, since we have

$$H_2(\mathbb{RP}^{8-p}, \widetilde{\mathbb{Z}}) = \begin{cases} \mathbb{Z}_2 & p < 6 \\ \mathbb{Z} & p = 6\,. \end{cases} \tag{4.1}$$

To confirm this claim, we have to show the wrapped D$(p+2)$-brane changes the discrete torsion of orientifold p-plane associated with the R-R $(5-p)$-form field. We can prove this in a similar way as in [1], in which arguments for O3-planes with spherical five-branes are given. Let us consider a spherical D$(p+2)$-brane with a radius r_0 wrapped around an Op-plane. The discrete torsion is defined by

$$I(r) = \frac{1}{2\pi}\oint_{\mathbb{RP}^{5-p}} C_{5-p} \mod 1\,, \tag{4.2}$$

where r is a radius of the $\mathbb{RP}^{5-p}$. If r is much larger than r_0, this integral gives the discrete torsion of the whole system of the Op-plane and the spherical D$(p+2)$-brane, while in the $r \to 0$ limit, it gives that of only the Op-plane.

In the covering space, the D$(p+2)$-brane is wrapped on S^2 with radius r_0. Let us think about a process in which r changes from 0 to infinity. In this process, S^{5-p} sweeps a $(6-p)$-dimensional manifold. If we compactify this manifold by adding a point at infinity, it becomes S^{6-p} as depicted in figure 2.

Because this S^{6-p} and the S^2, on which the D$(p+2)$-brane is wrapped, are linked with each other in a $(9-p)$-dimensional space perpendicular to the Op-plane, the following integral picks up the R-R charge of the D$(p+2)$-brane.

$$\frac{1}{2\pi}\oint_{S^{6-p}} dC_{5-p} = 1\,. \tag{4.3}$$

By Stokes' theorem, we obtain

$$2I(r=\infty) - 2I(r=0) = 1\,. \tag{4.4}$$

(We need the factor 2 in the left-hand side because we are discussing in the covering space.) This implies the wrapped D$(p+2)$-brane changes the discrete torsion.

Moreover, we have to show that the wrapped D$(p+2)$-brane changes the R-R charge of the Op-plane correctly. It should change the R-R charge of Op^- by $1/2$ and should not change that of Op^+. Therefore, the R-R Dp-brane charge which the wrapped D$(p+2)$-brane carries should be

$$Q = \frac{1}{2} + I_{\rm NS} \quad \mod 1\,, \tag{4.5}$$

where $I_{\rm NS}$ is the discrete torsion of the Op-plane associated with the NS-NS B_2 field,

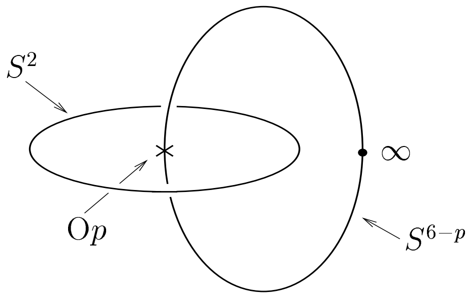

Figure 2: S^{6-p} and S^2 are linked with each other.

$$I_{\rm NS} = \frac{1}{2\pi}\oint_{\mathbb{RP}^2} B_2 \quad \mod 1\,. \tag{4.6}$$

This is explained as follows. The action on the D$(p+2)$-brane involves a term

$$\frac{1}{2\pi}\int_{p+3} (f_2 + B_2)\wedge C_{p+1}\,, \tag{4.7}$$

where f_2, B_2 and C_{p+1} are the field strength of the gauge field on the D$(p+2)$-brane, the NS-NS 2-form field and the R-R $(p+1)$-form field respectively. We can easily see that the coupling between B_2 and C_{p+1} reproduces the second term in (4.5). Taking account of the shifted quantization condition (3.11), we can understand that the $1/2$ term in (4.5) is provided by the coupling $f_2 \wedge C_{p+1}$.

Next, we examine Op-planes with $p \le 5$ enclosed by NS5-branes. A similar argument as above implies that if we wrap an NS5-brane on $\mathbb{RP}^{5-p}$ enclosing an Op-plane, the discrete torsion associated with the NS-NS 2-form field changes. Therefore, it is reasonable to claim the following.

> *An NS5-brane wrapped on $\mathbb{RP}^{5-p}$ changes an Op^--plane ($\widetilde{\rm O\mathit{p}}^{\,-}$-plane) to an O$p^+$-plane ($\widetilde{\rm O\mathit{p}}^{\,+}$-plane), and vice versa.*

The Dp-brane charge of an Op^+-plane and that of $\widetilde{\rm O\mathit{p}}^{\,+}$-plane are both $+2^{p-5}$. Hence the wrapped NS5-brane should carry the R-R charge of

$$Q = 2^{p-4} + I_R\,, \tag{4.8}$$

$$
\begin{tikzcd}[column sep=huge, row sep=large]
\mathrm{O}p^{-} \arrow[r, leftrightarrow, "\mathbb{RP}^2\ \mathrm{D}(p+2)"] \arrow[d, leftrightarrow, "\mathbb{RP}^{5-p}\ \mathrm{NS5}"'] & \widetilde{\mathrm{O}p}^{-} \arrow[d, leftrightarrow, "\mathbb{RP}^{5-p}\ \mathrm{NS5}"] \\
\mathrm{O}p^{+} \arrow[r, leftrightarrow, "\mathbb{RP}^2\ \mathrm{D}(p+2)"'] & \widetilde{\mathrm{O}p}^{+}
\end{tikzcd}
$$

Table 2: The relations between the types of orientifold planes and the wrapped branes.

where I_R is the discrete torsion associated with C_{5-p}, which is defined by

$$I_R = \frac{1}{2\pi}\oint_{\mathbb{RP}^{5-p}} C_{5-p} \quad \mod 1\,. \tag{4.9}$$

The I_R term in (4.8) is easily reproduced in the same way as we did above for the wrapped D-brane. The NS5-brane action has the following term.

$$\frac{1}{2\pi}\sum_p \int_6 (h_{5-p} + C_{5-p}) \wedge C_{p+1}\,, \tag{4.10}$$

where h_{5-p} are the field strengths of the gauge fields on the NS5-brane. For type-IIA (-IIB) NS5-branes, the sum is taken over $p = 0, 2, 4$ ($p = -1, 1, 3, 5$). The $p = 3$ term is S-dual of (4.7) and other terms are required by the consistency with the T-duality. The second term in (4.8) is generated by the coupling between C_{5-p} and C_{p+1} in (4.10). The first term in (4.8) is explained by supposing the flux quantization

$$\frac{1}{2\pi}\oint_{\mathbb{RP}^{5-p}} h_{5-p} \in \mathbb{Z} + 2^{p-4}\,. \tag{4.11}$$

We do not know how to prove this quantization condition directly, but we can see this condition is consistent with duality in some specific cases. For the case of $p = 3$, as expected by S-duality, this is the same condition as (3.11). The condition for the case of $p = 4$ is consistent with the analyses of the M-theory lift of O4-planes given in [2, 17].

In the case of O0-planes and O1-planes, we can consider two extra $\mathbb{Z}_2$ discrete torsions which are associated with the R-R $(1-p)$-form field and the NS-NS 6-form field, respectively [3]. We can prove the following statement in the same way as above.

> *For Op-planes with $p \le 1$, we can change the discrete torsions with respect to the R-R $(1-p)$-form field and the NS-NS 6-form field by wrapping a D$(p+6)$-brane on $\mathbb{RP}^6$ and a fundamental string on $\mathbb{RP}^{1-p}$, respectively.*

O2-planes also have extra non-trivial cohomology group $H^0(\mathbb{RP}^6, \mathbb{Z}) = \mathbb{Z}$. It is not torsion but an integral charge. This charge is identified with the background cosmological constant just as the H^0-cohomology for O6-planes. In fact, for $p = 2$, the wrapped D-brane is a D8-brane, and this plays a role of a domain wall which divide the background into two regions with different cosmological constant.

5. Non-BPS D-branes and orientifold planes

In this section, we argue some relations between $\widetilde{\mathrm{O}p}^-$-planes and stable non-BPS D-branes in type-I string theory. We will interpret the results in the previous sections in terms of non-BPS D-branes.

To begin with, let us review stable non-BPS D-branes in type-I string theory in brief. As shown in [8], the Dp-brane charges in type-I string theory can be classified by the reduced K-theory group $\widetilde{\mathrm{KO}}(S^{9-p})$, or equivalently by the $(8-p)$th homotopy group of the perturbative gauge group O(32). The non-trivial homotopy groups of O(32) are given as follows,

$$\pi_7(\mathrm{O}(32)) = \pi_3(\mathrm{O}(32)) = \mathbb{Z}\,, \tag{5.1}$$

$$\pi_9(\mathrm{O}(32)) = \pi_8(\mathrm{O}(32)) = \pi_1(\mathrm{O}(32)) = \pi_0(\mathrm{O}(32)) = \mathbb{Z}_2\,. \tag{5.2}$$

The first line corresponds to BPS D1 and D5-brane charges and the second line corresponds to non-BPS D(−1), D0, D7 and D8-brane $\mathbb{Z}_2$ charges, respectively [8]. Note that the non-BPS D7 and D8-branes in type-I string theory are known to have some subtleties in the arguments of the stability [18, 19]. Namely, the open strings stretched between these objects and background D9-branes create tachyonic modes which may cause the instability. However, it will not affect our arguments below, since we only consider the topological structure of the system which will remain after the tachyon condensation.

Let us consider type-I string theory compactified on S^1. A non-BPS D8-brane can be constructed in the same way as the construction of a non-BPS D-particle using a half D1-$\overline{\mathrm{D1}}$[3] pair [7]. Consider a half D9-$\overline{\mathrm{D9}}$ pair wrapping the S^1, and assign the -1 Wilson line on the half D9-brane and the $+1$ Wilson line on the half $\overline{\mathrm{D9}}$-brane. This system is topologically equivalent to a non-BPS D8-brane. Adding background 16 D9-branes, we obtain the following Wilson line:

$$g_1 = \mathrm{diag}(-1, (+1)^{32})\,, \qquad g_1' = +1\,, \tag{5.3}$$

where g_1' denotes the Wilson line on the half $\overline{\mathrm{D9}}$-brane. This is equivalent to the Wilson line

$$g_1 = \mathrm{diag}\left(-1, +1, (+1)^{30}\right)\,, \tag{5.4}$$

[3]In this paper, we count the number of D-branes after the orientifold projection. Therefore, the gauge group on n D1-branes (or D9-branes) in type-I string theory is $\mathrm{O}(2n)$. For $n = 1/2$, the gauge group is $\mathrm{O}(1) = \mathbb{Z}_2 = \{\pm 1\}$.

up to creation and annihilation of D-brane anti D-brane pairs. This Wilson line actually picks up the non-trivial element of $\pi_0(\mathrm{O}(32)) = \mathbb{Z}_2$, which corresponds to the non-BPS D8-brane charge as mentioned above. We argued in section 2 that this Wilson line is not allowed, since it is not compatible with the existence of non-BPS D-particles. In other words, we must have even number of non-BPS D8-branes which are transverse to the S^1. We will come back to this point later, but now let us formally consider one non-BPS D8-brane, which can be realized in perturbative type-I string theory, since we would like to analyze each non-BPS D8-brane individually. Taking T-duality transformation along the S^1, we can show that a configuration with a non-BPS D9-brane stretched between two $\mathrm{O8}^-$-planes is equivalent to that with two $\widetilde{\mathrm{O8}}^-$-planes.

This can be generalized as follows. Let us compactify the x^1, $x^2, \ldots$ and x^{9-p} directions of type-I string theory on $\mathbb{T}^{9-p}$ and fix O(32) Wilson lines to arbitrary values. To introduce a non-BPS D8-brane, we add a half D9-brane and a half $\overline{\mathrm{D9}}$-brane with the following Wilson lines:

$$g_1 = -1\,, \qquad g_2 = \cdots = g_{9-p} = +1\,, \quad (\text{for D9}) \tag{5.5}$$

$$g_1' = g_2' = \cdots = g_{9-p}' = +1\,. \quad (\text{for } \overline{\mathrm{D9}}) \tag{5.6}$$

If we carry out the T-duality transformation before the tachyon condensation, these Wilson lines change the charge of an Op-plane at $(x^1, x^2, \ldots, x^{9-p}) = (+1, +1, \ldots, +1)$ by $-1/2$ and one at $(-1, +1, \ldots, +1)$ by $+1/2$. (We normalize the coordinate such that the orientifold planes are located at $x^i = \pm 1$.) On the other hand, if we take a T-duality transformation after the tachyon condensation, a non-BPS D8-brane is mapped to a non-BPS D$(p+1)$-brane stretched between the two orientifold planes (figure 3i). Therefore, we obtain the following relation.

> **Relation (i).** *A configuration with a non-BPS D$(p+1)$-brane stretched between two Op-planes is topologically equivalent to that with two opposite type Op-planes.*

Here we only consider $\mathrm{O}p^-$ and $\widetilde{\mathrm{O}p}^-$, and they are called the opposite type to each other in the above statement. This relation is essentially shown in [20] and used in the explanation of the brane transfer operation. Note that the non-BPS D$(p+1)$-brane above is unstable due to tachyonic modes on it. In usual situations in type-II string theory without orientifold planes, non-BPS D-branes do not have any conserved charge and they are believed to decay into the supersymmetric vacuum after the tachyon condensation. However, in our case, the non-BPS D$(p+1)$-brane changes the type of orientifold plane which the brane is attached on. This implies that non-BPS D$(p+1)$-branes carry a $\mathbb{Z}_2$ charge as well as type-I D8-branes.

Next, we would like to consider T-dual picture of type-I non-BPS D7-branes. Let us consider type-I string theory compactified on $\mathbb{T}^{9-p}$ and fix the O(32) Wilson lines

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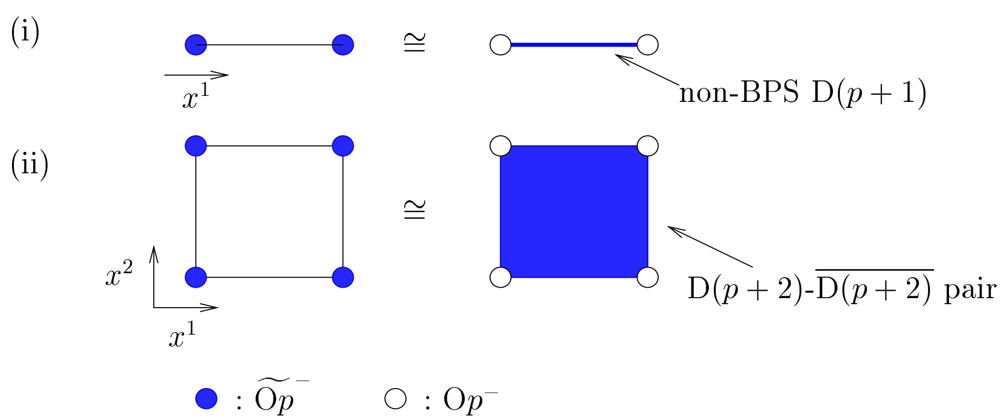

Figure 3: The relations (i) and (ii). The types of Op-planes are flipped after condensation of the tachyonic modes on the "non-BPS D-branes".

to arbitrary values again. A non-BPS D7-brane perpendicular to the x^1-x^2 plane is realized by adding two (four halves) D9-branes and two (four halves) $\overline{\mathrm{D9}}$-branes with the following Wilson lines:

$$\left.\begin{array}{c} g_1 = \mathrm{diag}(-1,-1,+1,+1)\,, \qquad g_2 = \mathrm{diag}(-1,+1,-1,+1)\,, \\ g_3 = \cdots = g_{9-p} = \mathrm{diag}((+1)^4)\,. \end{array}\right\} \quad (\text{for D9}) \tag{5.7}$$

$$g'_1 = g'_2 = \cdots = g'_{9-p} = \mathrm{diag}((+1)^4)\,. \quad (\text{for } \overline{\mathrm{D9}}) \tag{5.8}$$

The Wilson lines (5.7) are also argued to be unacceptable in section 2. Here, let us formally consider these Wilson lines as we did in the case of the non-BPS D8-brane above. If we carry out the T-duality transformation before taking account of the tachyon condensation, these extra Wilson lines change the charges of four Op-planes at $(\pm 1, \pm 1, (+1)^{7-p})$ by $-3/2$, $+1/2$, $+1/2$, $+1/2$, respectively. (The integral part of this charge assignment is not important because we can change them by moving Dp-branes from one Op-plane to another.) Therefore, the type of these four Op-planes are changed. On the other hand, if we first take the tachyon condensation on the D9-$\overline{\mathrm{D9}}$ system to obtain a non-BPS D7-brane perpendicular to the x^1-x^2 plane and then carry out the T-duality transformation for all directions on $\mathbb{T}^{9-p}$, we obtain one "non-BPS D$(p+2)$-brane" wrapped on a hyperplane containing the four Op-planes at $(\pm 1, \pm 1, (+1)^{7-p})$. Because a type-I non-BPS D7-brane can be constructed as a D7-$\overline{\mathrm{D7}}$ pair in type-IIB string theory projected by Ω, where Ω is the world-sheet parity operator exchanging D7 and $\overline{\mathrm{D7}}$ [8, 18], the "non-BPS D$(p+2)$-brane" is nothing but a pair of a D$(p+2)$ and a $\overline{\mathrm{D}(p+2)}$. Hence we conclude that (figure 3*ii*):

> **Relation (ii).** *A configuration with a D$(p+2)$-$\overline{\mathrm{D}(p+2)}$ pair stretched among four Op-planes is topologically equivalent to that with four opposite type Op-planes.*

Note again that the D$(p+2)$-$\overline{\mathrm{D}(p+2)}$ pair has tachyonic modes apart from the Op^--planes. After condensation of these tachyonic modes, the types of four Op-planes on the "non-BPS D$(p+2)$-brane" are flipped. This reflects the fact that the "non-BPS D$(p+2)$-brane" carries a $\mathbb{Z}_2$ charge, which corresponds to the non-BPS D7-brane charge in type-I string theory.

In addition to these T-duality relations among non-BPS configurations, they are related by "the descent relation" [21, 7], which relates D-branes with different dimensions via tachyon condensations. On the type-I side, we can construct a non-BPS D7-brane using two non-BPS D8-branes. Let us compactify type-I string theory on S^1 and consider two non-BPS D8-branes wrapped on the S^1. Since the gauge group on a non-BPS D8-brane is $\mathbb{Z}_2$ [20], we can assign different Wilson lines for two non-BPS D8-branes. Due to these Wilson lines, tachyon condensation generates a kink solution and it is identified with a non-BPS D7-brane. On type-II side, we can construct non-BPS D$(p+1)$-branes stretched between two Op-planes from a D$(p+2)$-$\overline{\mathrm{D}(p+2)}$ pair. Let us consider a D$(p+2)$-$\overline{\mathrm{D}(p+2)}$ pair wrapped on a hyperplane containing four Op-planes at $(\pm 1, \pm 1, (+1)^{7-p})$. There are two real tachyon fields on the D$(p+2)$-$\overline{\mathrm{D}(p+2)}$ pair. Let $T(x^2)$ denote a mode of one of these tachyon fields depending only on x^2. Because the tachyon field changes its sign by the orientifold flip, $T(x^2)$ cannot have an expectation value at $x^2 = \pm 1$. Therefore, as a result of the condensation of the T-field, we obtain a pair of kink solutions at $x^2 = \pm 1$. These are identified with two non-BPS D$(p+1)$-branes, which are T-dual to the two non-BPS D8-branes used to construct a non-BPS D7-brane. Similarly, we can proceed one more step. There is a real tachyon field on the non-BPS D$(p+1)$-brane, whose sign is flipped under the orientifold flip. Hence, we obtain a pair of kink solutions at the locations of the two Op-planes, $x^1 = \pm 1$, as above. These are now identified with two 1/2 Dp-branes stuck on the two Op-planes.

Now let us consider type-I string theory compactified on $\mathbb{T}^3$ which is parameterized by x^1,x^2 and x^3, and investigate more detailed structure. We study the dual $\mathbb{T}^3/\mathbb{Z}_2$ orientifold for simplicity, but most of the results below can easily be generalized to $\mathbb{T}^n/\mathbb{Z}_2$ orientifolds.

As shown in [8, 20], the gauge group of a type-I non-BPS D7-brane is U(1). Let us consider a non-BPS D7-brane which is perpendicular to the x^1 and x^2 direction. Taking T-duality along the x^1, x^2 and x^3 directions, we obtain a D8-$\overline{\mathrm{D8}}$ pair which is localized in the x^3 direction and extended in the x^1 and x^2 directions. The degree of freedom of a U(1) Wilson line along x^3 on the non-BPS D7-brane implies that the D8-$\overline{\mathrm{D8}}$ pair in the T-dualized picture can split and move in the x^3 direction. Therefore, we obtain a picture that four O6$^-$-planes are caught between a D8-brane and a $\overline{\mathrm{D8}}$-brane in the covering space. (Figure 4*a*)

Similarly, a non-BPS D7-brane stretched between two O6$^-$-planes can be interpreted as a configuration with two O6$^-$-planes cylindrically wrapped by a D8-brane. (Figure 4*b*.) In fact, the non-BPS D7-brane can be realized as a kink configuration

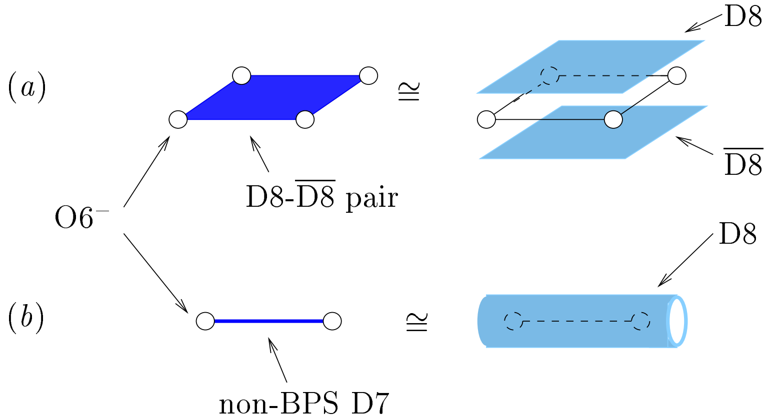

Figure 4: Descriptions of the "non-BPS D-branes" in $\mathbb{T}^3/\mathbb{Z}_2$ orientifold using D8-branes. The outside region divided by the D8-branes is in the background with odd cosmological constant.

of a D8-$\overline{\text{D8}}$ pair, in which the tachyons created by the D8-$\overline{\text{D8}}$ strings are condensed apart from a hyperplane of codimension one stretched between two O6$^-$-planes. If we identify the tachyon condensation with the pair annihilation of the D8-$\overline{\text{D8}}$ pair, we actually obtain the cylindrical configuration.

Using this re-interpretation of non-BPS D-branes by D8-branes, we can easily explain why these non-BPS D-branes change the type of O6-planes when they decay. The D8-branes in figure 4 can be deformed into spherical D8-branes enclosing O6-planes. According to the argument in section 4, these spherical D8-branes change the type of O6-planes.

Furthermore, we can argue the shifted quantization condition (3.11) in terms of the tachyon condensation. Consider an O6-plane located at $x^1 = x^2 = x^3 = 0$ and a D8-$\overline{\text{D8}}$ pair on top of it, localized in the x^3 direction. There is a complex tachyon field denoted again by T on the world-volume of the D8-$\overline{\text{D8}}$ pair. We consider the tachyon field which depends only on x^1 and x^2. As noted above, this tachyon field changes its sign by the orientifold flip and hence satisfies

$$T\left(-x^1, -x^2\right) = -T\left(x^1, x^2\right) \tag{5.9}$$

in the covering space. Note that this equation implies $T(0,0) = 0$. Suppose that the tachyon condenses apart from the O6-plane, namely, $T(x^1, x^2) \neq 0$ for all $(x^1, x^2) \neq (0,0)$. Then it is easy to show using (5.9) that it makes a vortex of odd winding number. This implies that it behaves as an odd number of half D6-branes. If we think that the D8-$\overline{\text{D8}}$ pair is annihilated apart from the O6-plane due to the tachyon condensation, we will have a configuration with the O6-plane wrapped by a spherical D8-brane. Therefore, what we have shown here is nothing but the shifted quantization condition (3.11).

Furthermore, we can easily select allowed configurations including non-BPS D7-branes and/or D8-$\overline{\text{D8}}$ pairs with a help of the re-interpretation. A D8-brane changes

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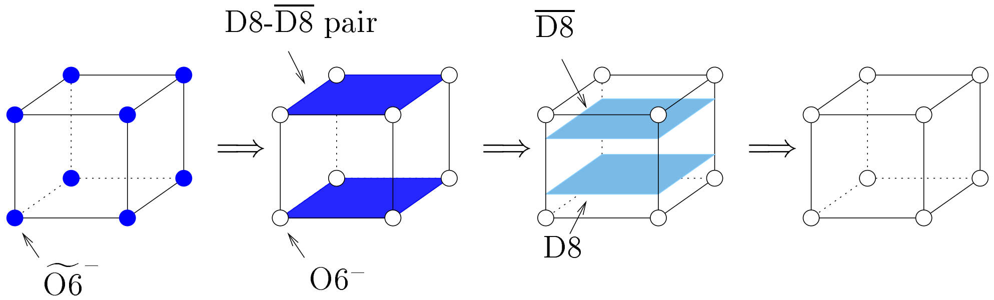

Figure 5: Transfer of stuck D-branes. The system with eight $\widetilde{\text{O6}}^-$-planes can be continuously deformed to that with eight O6$^-$-planes.

the background cosmological constant by one unit. Therefore, if we accept the fact suggested in section 3 that O6$^-$($\widetilde{\text{O6}}^-$)-planes are allowed only in the background with even (odd) cosmological constant, we can easily show that the configurations associated with the Wilson lines (5.4) and (5.7) are not allowed. The allowed configurations can always be transformed to the configuration with eight O6$^-$-planes or that with eight $\widetilde{\text{O6}}^-$-planes using relations (i) and (ii).

Finally, we would like to point out another phenomenon concerned with the transfer of stuck D-branes. Let us consider type-I string theory compactified on $\mathbb{T}^3$ with Wilson lines (2.4), and take T-duality along all three directions of the torus. Namely we consider $\mathbb{T}^3/\mathbb{Z}_2$ orientifold with eight $\widetilde{\text{O6}}^-$-planes. Using the relation (ii), we know that this system is topologically equivalent to that with eight O6$^-$-planes and two D8-$\overline{\text{D8}}$ pairs, as depicted in figure 3*ii*. Now, recall that the D8-$\overline{\text{D8}}$ pair can split and move along the x^3 direction. If we move one of the D8-$\overline{\text{D8}}$ pairs to coincide with the other, they will annihilate completely and only eight O6$^-$-planes remain. (Figure 5.) Namely, the system with eight $\widetilde{\text{O6}}^-$-branes can be continuously deformed to that with eight O6$^-$-branes! We will examine this process more carefully in the next section.

6. Transfer of stuck D-branes

In section 4 we showed that stuck Dp-branes on Op-planes are not really stuck but can be moved away from Op-planes as magnetic flux on D$(p+2)$-branes. Using this, we can continuously deform an orientifold with eight O6$^-$-planes to that with eight $\widetilde{\text{O6}}^-$-planes as mentioned in the previous section. In the context of $\mathbb{T}^3$ compactified type-I string theory, this implies the trivial Wilson lines can be continuously deformed into the Wilson lines (2.4). The purpose of this section is to show that this is actually possible and that the process is actually related to a deformation of D8-branes in $\mathbb{T}^3/\mathbb{Z}_2$ orientifold via T-duality.

Let us consider Wilson lines of gauge group $G = \mathrm{Spin}(8)$ on $\mathbb{T}^3$. We will use only Spin(8) subgroup of $\mathrm{Spin}(32)/\mathbb{Z}_2$ associated with four D-branes (and their mirror images), and other D-branes will be omitted. Let us start from trivial Wilson lines $g_1 = g_2 = g_3 = 1$. We can continuously deform them to the following Wilson lines keeping it a zero-energy configuration:

$$\begin{aligned} g_1 &= \mathrm{diag}(+1,-1,+1,-1,+1,-1,+1,-1)\,, \\ g_2 &= \mathrm{diag}(+1,+1,-1,-1,+1,+1,-1,-1)\,, \\ g_3 &= \mathrm{diag}(+1,+1,+1,+1,+1,+1,+1,+1)\,. \end{aligned} \tag{6.1}$$

The final configuration we want to realize by continuous deformation is

$$\begin{aligned} g_1 &= \mathrm{diag}(+1,-1,+1,-1,+1,-1,+1,-1)\,, \\ g_2 &= \mathrm{diag}(+1,+1,-1,-1,+1,+1,-1,-1)\,, \\ g_3 &= \mathrm{diag}(+1,+1,+1,+1,-1,-1,-1,-1)\,. \end{aligned} \tag{6.2}$$

To get the final configuration (6.2) we have to change g_3 in (6.1) to that in (6.2). We cannot do this without raising the energy. Let us parameterize g_3 between (6.1) and (6.2) by $0 \le \alpha \le 1$. The final value $g_3(\alpha = 1)$ breaks the gauge group G into

$$H = \mathrm{Spin}(4)^2/\mathbb{Z}_2 = (\mathrm{Spin}(4) \times \mathrm{SU}(2) \times \mathrm{SU}(2))/\mathbb{Z}_2\,. \tag{6.3}$$

In this subgroup, $g_3(1)$ is represented as $g_3(1) = \mathbf{1}_4 \otimes \mathbf{1}_2 \otimes -\mathbf{1}_2$. Let us take the following path between $\alpha = 0$ and 1.

$$g_3(\alpha) = \mathbf{1}_4 \otimes \mathbf{1}_2 \otimes e^{\pi i \alpha \sigma_z}\,. \tag{6.4}$$

At an interpolating value of α, the gauge group is further broken into

$$H' = (\mathrm{Spin}(4) \times \mathrm{SU}(2) \times \mathrm{U}(1))/\mathbb{Z}_2\,. \tag{6.5}$$

This subgroup does not contain g_1 and g_2 as its elements. So, when we turn on the parameter α, the gauge configuration on the x^1-x^2 plane must be deformed such that it is embedded in the subgroup H'.

We can determine the gauge configuration realized after turning on α as follows. Let us regard g_1 and g_2 as elements of H. The non-trivial homotopy $\pi_1(H) = \mathbb{Z}_2$ admits Dirac strings, and g_1 and g_2 represent a gauge configuration with one Dirac string. Even if the gauge group is broken to H', the Dirac string should remain. More precisely, $\pi_1(H') = \mathbb{Z}$ due to the U(1) factor and the number of Dirac strings of gauge group H' should be an odd integer. Unlike SU(2) case, gauge configuration with U(1) Dirac strings cannot be realized as a vacuum configuration. The Dirac strings must always be accompanied by the same number of magnetic flux. (For example, the magnetic charge of a Dirac monopole is equal to the number of Dirac strings going out from the monopole.) Therefore, when $0 < \alpha < 1$, non-zero magnetic flux associated with the U(1) factor of H' is generated.

When α reach the final value 1, the gauge group H is restored and U(1) factor is enhanced to SU(2) again. At the same time, the magnetic flux vanishes to leave the Wilson lines g_1 and g_2 again and finally we get the Wilson lines (6.2).

Next, let us discuss T-dual of this process. Before going to $\mathbb{T}^3/\mathbb{Z}_2$ orientifold, it is convenient to insert one step. By taking T-duality only along the x^3-direction for type-I string theory, we have $S^1/\mathbb{Z}_2 \times \mathbb{T}^2$ orientifold in type-IIA string theory. This contains four D8-branes (and 12 we are now omitting). At first, when $\alpha = 0$, these D8-branes stay on one of two O8-planes. The gauge group $G = \mathrm{Spin}(8)$ is realized on this O8-plane. Turning on the parameter α corresponds to moving two of these D8-branes from the O8-plane. Each factor of the broken gauge group $H' \sim \mathrm{Spin}(4) \times \mathrm{U}(2)$ represent the gauge symmetries on two D8-branes staying on the O8-plane and two moving D8-branes, respectively.

When $\alpha \neq 0$, the non-zero magnetic flux in type-I configuration is mapped into magnetic flux on the moving D8-branes. The magnetic flux on the D8-branes can be regarded as D6-branes absorbed in the world-volume of the D8-branes, and by checking the amount of the flux carefully we can find the number of the D6-branes is odd integer.

By taking further T-duality along the x^1 and x^2 directions for this configuration, we have a dual configuration with odd number of D8-branes wrapped along x^1 and x^2 directions and they hold two D6-branes on them. This actually shows that stuck D6-branes are transferred as magnetic flux on the D8-branes. The emergence and annihilation of the U(1) magnetic flux correspond to the pair creation and pair annihilation of D8-branes and $\overline{\mathrm{D8}}$-branes, which are mirror images of the D8-branes. This is the same relation with what used in ([22]) to analyze brane annihilation in terms of matrix theory.

7. Conclusion and discussion

In sections 2 and 3 we focused on Wilson lines in type-I string theory with vector structure and showed that the constraints for the Wilson lines are closely related to the discrete torsion of Op-planes associated with the R-R $(5-p)$ form field. It would be an interesting problem to extend this analysis to Wilson lines without vector structure. It was partially done in [23] up to $\mathbb{T}^3$ compactification. Because it is known that such Wilson lines are related with Op^+ and $\widetilde{\mathrm{O}p}^+$-planes [9], if we can obtain such a set of constraints for general Wilson lines, we would obtain more information about properties of Op^+ and $\widetilde{\mathrm{O}p}^+$. Furthermore, it is also interesting to consider a relation between type-I Wilson lines and extra non-trivial cohomologies for lower-dimensional orientifold planes.

In section 4 we showed wrapped branes change the discrete torsions of orientifold planes. To explain the change of the R-R charges of the orientifold planes, we used the shifted flux quantization conditions on the branes wrapped around the orientifold planes. As explained in section 5, we obtained an interesting derivation of the condition (3.11), but this is not the end of the story. We do not have any proof of the condition (4.11) as well as the explanation for the fractional charges of orientifold planes, which may be closely related to the quantization condition on the wrapped branes. Instead, we just notice that we cannot naively conclude they are inconsistent with the Dirac's quantization condition. If one want to obtain the quantization condition, one should use a brane coupling to the gauge flux. Deforming the brane along a closed path in the configuration space and demanding two partition functions for initial and final brane configurations to coincide, the Dirac's condition for the background gauge flux is obtained. In string theory, however, this is not a simple task because such branes are accompanied by dynamical fields on it. To obtain the correct partition functions, we should trace how these fields on branes vary during the brane deformation. Recently, such analysis was done for some cases in several works [16, 24] and similar approach is expected to solve the problem about the fractional charges of orientifold planes and shifted quantization conditions.

In this paper, we have not paid much attention to the supersymmetry. Because our analysis relies on topological aspects, we expect our arguments are reliable in non-supersymmetric cases. However, it is worth asking whether each Wilson line can be realized as a supersymmetric configuration. Concerning this question, there is one subtlety. In section 2 we required the commutativity of type-I Wilson lines. In ordinary Yang-Mills theory, this requirement guarantees the vanishing field strength and the vanishing energy. In supergravity, however, this is not sufficient to obtain a zero-energy configuration because the field strength of the two-form field in type-I string theory contains the Chern-Simons invariant.

$$H_3 = dB_2 - \omega_3 \,. \tag{7.1}$$

Even in the cases of vanishing Yang-Mills field strength, the Chern-Simons invariant does not always vanish. If it takes some fractional value, it cannot be canceled by the dB_2 term.[4] Actually, the Wilson lines (2.4) gives $\int \omega_3 \in \mathbb{Z}+1/2$ (The integral part is not determined even if we specify the Wilson lines because $\int \omega_3$ is not invariant under large gauge transformations.) and we cannot cancel this by $\int dB_2$, which is always integer. This corresponds to the fact that the configuration with eight $\widetilde{\mathrm{O6}}^-$-planes has odd cosmological constant. Via T-duality, H_3 is related with the cosmological constant Λ on the $\mathbb{T}^3/\mathbb{Z}_2$ orientifold by the equation,

$$\int H_3 = \frac{1}{2}\Lambda \,. \tag{7.2}$$

[4] Y.I. thanks J. deBore and K. Hori for pointing out this fact.

Notice that the normalization of the H_3-flux differs from that of R-R fluxes in type-II string theory by factor 2 because type-I D1-branes are fractional branes stuck on an $O9^-$-plane. Therefore, we cannot realize the Wilson lines (2.4) on a flat spacetime. This, however, does not implies inconsistency of the theory unlike anomalies. It just means the lower-dimensional theory obtained by the compactification has non-zero cosmological constant. Indeed, we can make a static and even supersymmetric classical solution like the type-I' configuration.

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